Frequency Prediction of a "Smart" Beam Using Eshelby Techniques with Linearly Distributed Strains

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ABSTRACT

This paper presents the interactions of a simple vibrating beam host with embedded arrays of devices (actuators/sensors). The geometry of the device is idealized to be ellipsoidal and Eshelby's classical techniques are used to obtain a first order estimate of the apparent stiffening of the host structure due to the interaction between the actuator and the surrounding host, as a result of both far field and actuation loads. The far-field load is assumed to have linear spatial dependence, while the actuation loads are assumed to be spatially uniform. Preliminary analytical results for piezoelectric actuators embedded in an isotropic host suggest that the solution for a uniformly distributed loading may provide adequate accuracy for small volume fraction of embedded devices.

INTRODUCTION

This paper illustrates the role of closed-form eigenstrain methods based on Eshelby's equivalent inclusion eigenstrain techniques [1,2], for quantifying the mechanical interactions in a sample Euler-Bernoulli beam with rows of embedded equally spaced piezoelectrical sensor/actuator devices. The equations of motion of the system are solved using Hamilton's principle, based on a variational formulation of the system under investigation. The Rayleigh quotient is used to study the change in the natural frequency of the structure due to harmonic excitation of the actuators.

As shown schematically in Figure 1, two rows of uniformly spaced devices are embedded in the beam at a distance d/2 symmetrically about the neutral plane of the beam. Both rows of devices are excited simultaneously, in order to control the vibrational characteristics of the beam. As the beam flexes, every alternate device in each row acts as a sensor and the outputs are used in a closed-loop feedback circuit to actuate the active half of the opposite row of devices. The actuation strain is assumed to be opposed in sign to the bending strain for all actuations. The influence of the devices on the vibrational

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bending strain for all actuations. The influence of the devices on the vibrational characteristics is investigated analytically by modeling the mechanical interactions between each device and the host. The aim is to generate the electro-mechanical interaction information, necessary for combining the device response with that of the host beam, in an integrated dynamical equation of the adaptive structure. As an illustrative example, the actuators are given a harmonic excitation proportional (but opposite in sign) to the bending strain in the beam. The result is an apparent stiffening of the beam and an accompanying increase in the natural frequency ω.

Several simplifying assumptions are made in this approximate analytical study. Euler-Bernoulli beam theory is assumed to apply. Each embedded device is assumed to be a piezoelectric cylinder of elliptical cross-section, whose polarization axis is oriented along the length of the beam. Thus, each device is approximated to act like elastic heterogeneity embedded in a host structure. Perfect bonding is assumed at the interfaces. The sensor/actuator material is assumed to be PZT-5H, and the host material is assumed to be ALPLEX plastic. All materials are approximated to be linear and mechanically isotropic and all losses are ignored. The linearizing assumption limits the validity of this approximate analysis to small excitation voltages and small deformations. The assumption of mechanical isotropy is an acceptable approximation for most PZT materials.

ANALYSIS

Eshelby's classical equivalent-inclusion technique [1] is applied to obtain the elastic interaction fields, both in the device and in the host, under both uniform and linear external applied loads and under uniform internal actuation loads. External loads are handled through Eshelby's fictitious eigenstrain technique for uniform and linear loadings. Internal actuation loads are treated as a real eigenstrains and are obtained from the linearized, isothermal, coupled electro-mechanical constitutive models given below. The difference between PZT sensors and actuators in the present analytical context is that the sensor only has a fictitious eigenstrain due to external loads, while the actuator has both fictitious and real eigenstrains.

The linearized, isothermal, coupled electro-mechanical constitutive model is [3];

$$\vec{\sigma} = \underline{C}\,\vec{\epsilon} - \underline{h}^T \vec{E}_e \qquad (1)$$

$$\vec{D} = \underline{h}\,\vec{\epsilon} + \underline{e}\,\vec{E}_e$$

where $\vec{\sigma}$ is the mechanical stress vector, $\vec{\epsilon}$ is the total strain vector including mechanical as well as electro-mechanical contributions, E_e is the electrical field vector, and $\vec{D}$ is the electrical displacement vector. $\underline{C}$ is the mechanical stiffness tensor, $\underline{h}$ is the piezoelectric coupling tensor indicating the stress caused by completely constrained excitation of the PZT material under a unit applied electrical field, and $\underline{e}$ is the fully constrained dielectric tensor for the PZT material. Arrows over a quantity are used to denote vector quantities, while an underscore is used to denote tensor quantities. Clearly, only the mechanical portion of this constitutive model applies to the ALPLEX host material.

fictitious eigenstrain which has the same stress field as the real heterogeneity, under both external loads and internal actuation strains. Thus, in the heterogeneity:

$$\begin{aligned} \vec{\sigma}^{o} + \vec{\sigma}^{/} &= \underline{C}^{D} \left(\vec{\epsilon}^{o} + \vec{\epsilon}^{/} - \vec{\epsilon}^{r}\right) \\ &= \underline{C}^{H} \left(\vec{\epsilon}^{o} + \vec{\epsilon}^{/} - \vec{\epsilon}^{*}\right) \end{aligned} \tag{2}$$

where $\vec{\epsilon}^{*} = \vec{\epsilon}^{r} + \vec{\epsilon}^{f}$

$\underline{C}$ is the material stiffness tensor; superscripts D and H on the stiffness indicate the PZT device and the ALPLEX host, respectively; superscripts o, / , r, f and * on the stress and strain terms indicate applied far-field value, perturbation due to the presence of the heterogeneity, real actuation eigenstrains, fictitious eigenstrains due to external loading, and total eigenstrains, respectively. The far-field strain is assumed to consist of a uniform and linear contributions such that:

$$\epsilon^{o}_{ij} = E^{o}_{ij} + B^{o}_{ijk} X_{k} \tag{3}$$

Thus, the eigenstrain $\vec{\epsilon}^{*}$ is given as :

$$\epsilon^{*}_{ij} = E^{*}_{ij} + B^{*}_{ijk} X_{k} \tag{4}$$

and the perturbation strain $\epsilon^{/}$ is

$$\epsilon^{/}_{ij} = E^{/}_{ij} + B^{/}_{ijk} X_{k} \tag{5}$$

The eigenstrain $\vec{\epsilon}^{*}$ consists of $\vec{\epsilon}^{r}$ and $\vec{\epsilon}^{f}$. Thus

$$\epsilon^{r}_{ij} = E^{r}_{ij} + B^{r}_{ijk} X_{k} \tag{6}$$

$$\epsilon^{f}_{ij} = E^{f}_{ij} + B^{f}_{ijk} X_{k} \tag{7}$$

In this study, B^{r}_{ijk} is assumed to be a null tensor. In other words, $\vec{\epsilon}^{r}$ is uniformly distributed. Thus, the real actuation eigenstrain is obtained from Equation (1) as:

$$\vec{\epsilon}^{r} = \vec{E}^{r} = \underline{d}^{T} \vec{E}_{e} \tag{8}$$

$$\text{where} \qquad \underline{d} = \underline{h}\, \underline{S}^{D}$$

$\underline{d}$ represents the free-expansion of the piezoelectric actuator for a unit applied electric field and $\underline{S}^{D}$ is the compliance tensor of the device material.

The total eigenstrain is now related to the disturbance strain by Eshelby's strain concentration tensors $\underline{S}^E$ and $\underline{D}^E$:

$$E^f_{ij} = S^E_{ijkl}\, E^*_{kl} = S^E_{ijkl}\, (E^r_{kl} + E^f_{kl}) \tag{9}$$

$$B^f_{ijk} = D^E_{ijklmn}\, B^*_{lmn} = D^E_{ijklmn}\, (B^r_{lmn} + B^f_{lmn}) \tag{10}$$

As indicated earlier, B^r_{ijk} is assumed to be zero in this paper.

Eshelby's strain concentration tensors can be computed for infinite spaces by using Kelvin's fundamental three-dimensional elasticity result for point loads; and for half-spaces by using appropriate Green's functions [4]. Explicit forms for Eshelby's tensor are readily available in the literature for embedded isotropic heterogeneities of ellipsoidal geometries. Solutions for anisotropic cases may be obtained numerically [4].

Substituting Equations (9) and (10) in Equation (2), we obtain:

$$\begin{aligned} C^D_{ijkl}\, (E^o_{kl} + S^E_{klmn}\, (E^f_{mn} + E^r_{mn}) \\ - E^r_{kl}) = C^H_{ijkl}\, (E^o_{kl} + \\ S^E_{klmn}\, (E^f_{mn} + E^r_{mn}) - E^f_{kl} - E^r_{kl}) \end{aligned} \tag{11}$$

$$\begin{aligned} C^D_{ijkl}\, (B^o_{kls} + D^E_{klspmn}\, (B^f_{pmn} + B^r_{pmn}) \\ - B^r_{kls}) = C^H_{ijkl}\, (B^o_{kls} + \\ D^E_{klspmn}\, (B^f_{pmn} + B^r_{pmn}) - B^f_{kls} - B^r_{kls}) \end{aligned} \tag{12}$$

Recalling that $\underline{B}^r$ is zero valued, Equations (11) and (12) can now be solved for the unknown fictitious eigenstrains E^f and $\underline{B}^f$ in terms of the applied external bending strains E^o and $\underline{B}^o$ and the applied real actuation eigenstrain E^r . This completes the solution for the total stress and strain fields within the device, which can now be determined from Equation (2). If the applied strain is uniform or linear, so is the fictitious eigenstrain. Nonuniform applied strains can be approximated by a polynomial series which yields a series solution for the fictitious eigenstrain [4]. In this paper, the far-field applied strain is assumed to be linear over the length scale of each embedded device, in view of the simplifying assumptions stated in the previous section.

The stress and strain fields outside the device, in the surrounding host structure are more difficult to determine by Eshelby's technique. Fortunately, the exterior field is not required when computing the energy of the system due to the actuation.

In this paper the mechanical and electrical energy terms are computed to obtain the stiffening of the structure under harmonic excitation, through a suitable variational scheme. The variational principle is a generalized form of Hamilton's principle, and may be written as [5]:

$$\delta\left[\int_{t_0}^{t_1}(L + W)\,dt\right] = 0 \tag{13}$$

where the Lagrangian function L is the difference between the kinetic energy T and the electric enthalpy H. The work term, W, includes the potential of all applied mechanical loads and the electrical charges. Thus, using the definition of electric enthalpy; and noting that $\vec{E}$ is restricted to devices, we can write

$$L = T - H$$

$$L = \frac{1}{2}\int_V (\omega^2 \rho \vec{u}^T \vec{u} - \vec{\epsilon}^T \underline{C}\, \vec{\epsilon})\, dV + \frac{1}{2}\int_\Omega (\vec{E}_e^T\, \underline{e}\, \vec{E}_e + 2\, \vec{\epsilon}^T \underline{h}^T\, \vec{E}_e)\, dV \tag{14}$$

where ω is the natural frequency, ρ is the density of host or device, $\vec{u}$ is the displacement field, and Ω is the volume of the devices.

The work term, W, is given as

$$W = \int_A (\vec{T}^T \vec{u} - Q\,\phi)\; dA \tag{15}$$

where $\vec{T}$ is the specified mechanical surface traction, Q is the specified surface charge density, ϕ is the resulting potential and A is the surface area.

After some manipulations, for stationary solutions, the Rayleigh quotient can be presented as [6]

$$\omega^2 = \frac{\int_V (\vec{\epsilon}^T \underline{C}\, \vec{\epsilon})\, dV + \int_\Omega (\vec{E}_e^T\, \underline{e}\, \vec{E}_e)\, dV}{\int_V \rho\, \vec{u}^T\, \vec{u}\, dV} \tag{16}$$

The resulting natural frequency due to the actuation loads is the focus of our interest in this paper, and results from (i) the linear flexural field of the beam, (ii) the passive perturbation of the bending field due to the presence of the devices, and (iii) the actuation field of the actuators.

The first term in the numerator of Equation (16) is termed the mechanical energy, and is given by [8]:

$$2U_{mech} = \int_V \vec{\epsilon}^{o^T} \underline{C}^H \vec{\epsilon}^o \, dV + \int_\Omega \vec{\epsilon}^{*^T} \underline{C}^H \underline{S}^E \vec{\epsilon}^* \, dV + \int_\Omega \vec{\epsilon}^{o^T} \underline{\Delta C} \vec{\epsilon}^o \, dV + 2\int_\Omega \vec{\epsilon}^{o^T} \underline{\Delta C} \underline{S}^E \vec{\epsilon}^* \, dV + \int_\Omega \vec{\epsilon}^{*^T} \underline{S}^{E^T} \underline{\Delta C} \underline{S}^E \vec{\epsilon}^{*^T} \, dV \tag{17}$$

where $\underline{\Delta C} = \underline{C}^D - \underline{C}^H$

The first term in Equation (17) is evaluated by integrating the uniform bending strain energy density throughout the domain V of the beam. The remaining terms represent energy due to the imaginary eigenstrains, real eigenstrains, and far field strains, and require evaluation only over the domain Ω of the devices. The terms with real eigenstrains represent electromechanical energy. The exterior solutions for the real and fictitious eigenstrain problems are not required in this study. After some algebraic manipulations, the mechanical energy is rewritten as:

$$2U_{mech} = \int_V \vec{\epsilon}^{o^T} \underline{C}^H \vec{\epsilon}^o \, dV + \int_\Omega \vec{\epsilon}^{o^T} \underline{\Delta C} \vec{\epsilon}^o \, dV + \int_\Omega \vec{E}^{*^T} \underline{C}^H \underline{S}^E \vec{E}^* \, dV + 2\int_\Omega \vec{E}^{o^T} \underline{\Delta C} \underline{S}^E \vec{E}^{*^T} \, dV + \int_\Omega \vec{E}^{*^T} \underline{S}^{E^T} \underline{\Delta C} \underline{S}^E \vec{E}^* \, dV + \int_\Omega \vec{X}^T \underline{B}^{*^T} \underline{C}^H \underline{D}^E \underline{B}^* \vec{X} \, dV + 2\int_\Omega \vec{X}^T \underline{B}^{o^T} \underline{\Delta C} \underline{D}^E \underline{B}^* \vec{X} \, dV + \int_\Omega \vec{X}^T \underline{B}^{*^T} \underline{D}^E \underline{\Delta C} \underline{D}^E \underline{B}^* \vec{X} \, dV \tag{18}$$

The first two terms in Equation (18) represent the energy of the homogeneous applied bending field. All other terms represent electromechanical coupling terms. The last three terms represent the contribution from the nonuniform (linearly distributed) term. In the unstiffened (unactuated) case, the real eigenstrain will be zero and will not contribute to the natural frequency. The uniform solution (obtained by ignoring the last three terms) was presented earlier [8], and the purpose of this paper is to examine the change in accuracy by including the nonuniform terms.

In order to perform the integrations in Equations (16) and (18), all that remains now is to assume explicit representations for the applied flexural strain field, and the actuation eigenstrain. For example, in the Rayleigh scheme for estimating the natural frequency of conservative systems, an approximate displacement field can be assumed. In this example, the approximate bending field is assumed to be harmonic in time and sinusoidal in space:

$$w = \sum_1^n a_n \, Sin(\omega_n t) \quad Sin\left(\frac{n\pi y}{L}\right) \tag{19}$$

where the y axis is oriented along the length of the beam, w is the transverse displacement in the z direction, ω_n and a_n are the natural frequency and amplitude, respectively, of the n^{the} mode, L is the length of the beam, and t is time. Only the fundamental mode (n=1) is of interest in this study. Thus the only non-zero term in the bending strain field $\vec{\epsilon}^o$ is ϵ_2^o, and is given as:

$$\epsilon_2^0 = z\,\frac{\pi^2}{L^2} a_1\, Sin(\omega_1 t)\;\; Sin(\frac{\pi y}{L}) \tag{20}$$

where, z is the distance of the device from the neutral axis of the beam. This strain field varies linearly in z direction, and is modeled by a piece-wise linear distribution in the y direction. The only non-zero component of the uniform actuation voltage vector is now E_{e_2} and is assumed to be proportional to the output of the sensory devices, and hence, to the bending strains. Thus E_{e_2} is written as:

$$E_{e_2} = E_{e_2}^{*}\; Sin(\omega_1 t)\;\; Sin(\frac{\pi y}{L}) \tag{21}$$

where the amplitude $E_{e_2}^{*}$ is selected to be proportional to the amplitude of the bending strain due to the fundamental vibrational mode of the beam:

$$E_{e_2}^{*} = K\, a_1\, z\, \frac{\pi^2}{L^2} \tag{22}$$

K is the feed-back proportionality constant.

The non-zero terms of the actuation strain vector are now written as:

$$\epsilon_i^I = d_{2i}\, E_{e_2} \quad , \quad i = 1\text{-}6 \tag{23}$$

Equations (19-23) are used in Equation (16) to compute the natural frequency of the system. The Rayleigh scheme for estimating the fundamental natural frequency of the structure is helpful to understand the dynamic behavior of the beam. Sample results, for natural frequency of the adaptive beam structure, are presented in the next section.

RESULTS AND DISCUSSIONS

The material properties assumed in this analysis for the PZT-5H sensor/actuator device and for the ALPLEX host are given in previous paper [8].

Figures (2)-(4) show the stiffening effect by assuming uniform and linear applied field. For convenience, the natural frequency (ω) is normalized with respect to that for zero electrical excitation (ω_0), and the actuation strain amplitude (ϵ^I) is normalized with respect to the far field strain (ϵ^0). The increase in the natural frequency is a measure of the stiffening of the beam due to the actuation loads.

The incremental change obtained by assuming a linear applied field is found to be negligible compared to the solution for the uniform field. For example, the maximum change is less than 1%, for a device volume fraction of 20%. Thus, these figures are almost identical as those obtained for the uniform solution [8]. In Figure (2), the volume fraction (V_f) of devices has been increased, not by increasing the number of devices as in previous papers [2,7,8], but by increasing the size of each device, relative to the host beam dimensions. The maximum volume fraction is maintained below 5 %, in order to minimize the interactions with neighboring devices and with boundaries of the host. Figure (3) illustrates the dependence of the stiffening effect (natural frequency) on the location of the actuators, relative to the neutral axis. As expected, moving the actuators away from the neutral axis increases the natural frequency, for the same excitation strain. Figure (4) illustrates the relative contributions of the mechanical interaction energy (U_{mech}), and the electrical energy (U_{elect}), towards stiffening of the structure. The mechanical term is comparable in magnitude to the electrical term, and its relative contribution increases as the actuation load or the host stiffness is increased.

CONCLUSIONS

This paper presents a unified approach, based on Eshelby's technique, for addressing the interaction mechanics of devices embedded in an adaptive structure. The analysis used is based on assumptions of linearly and uniformly distributed far field loadings and only uniform actuation strains. The results did not change significantly by assuming a linearly distributed applied field, and the maximum percentage change is less than 1%. Thus the first order approximation (uniform solution) is adequate for modeling the "smart" beam for low volume fractions of actuators. In view of the algebraic complexity, polynomial solutions of higher degree (i.e. quadratic and higher) are considered to be unnecessary for this type of problem.

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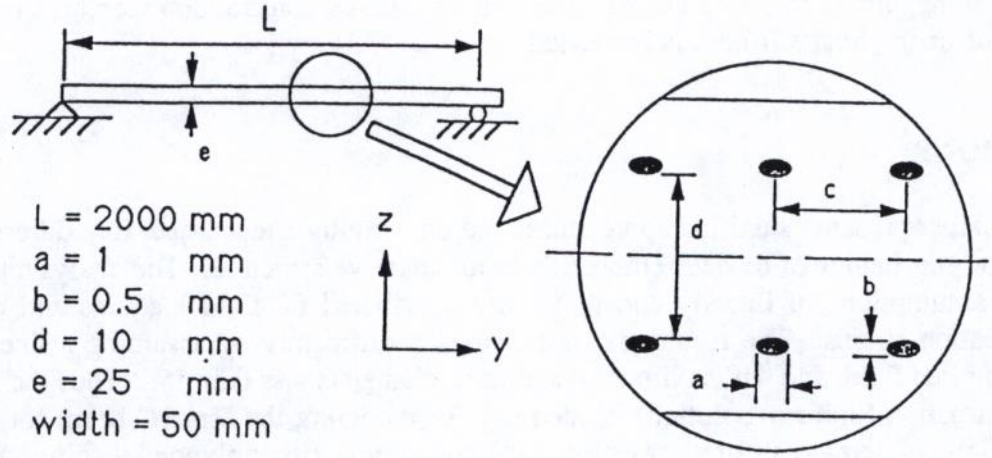

Figure 1. Schematic of Adaptive Beam with Embedded Rows of Devices.

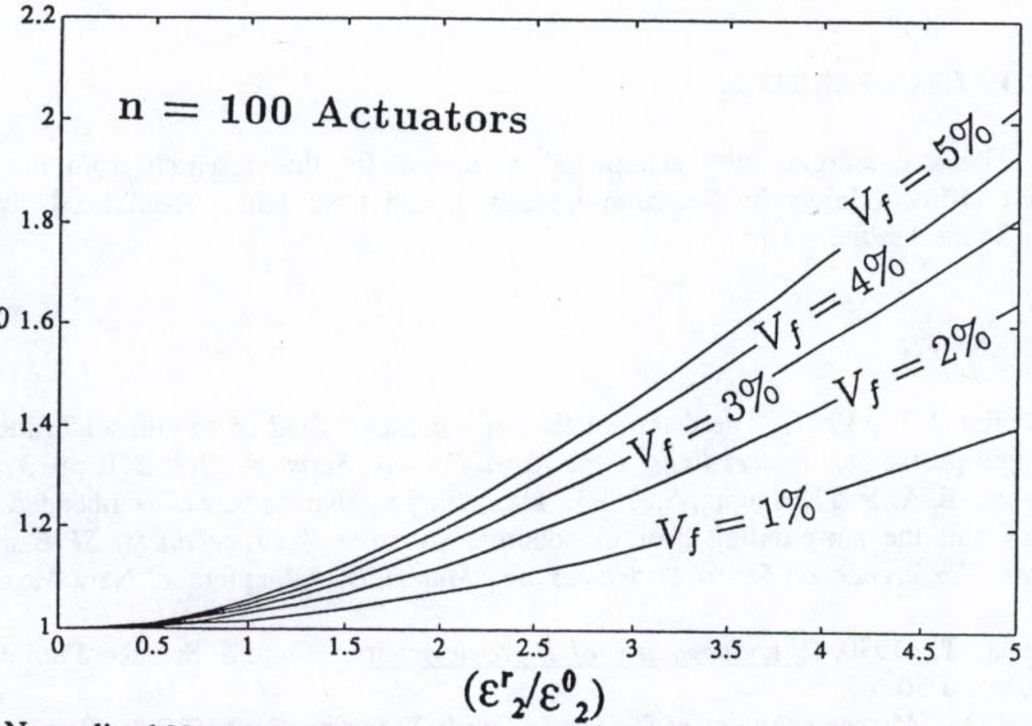

Figure 2. Normalized Natural Frequency as a function of Normalized Actuation Strain for different device densities.

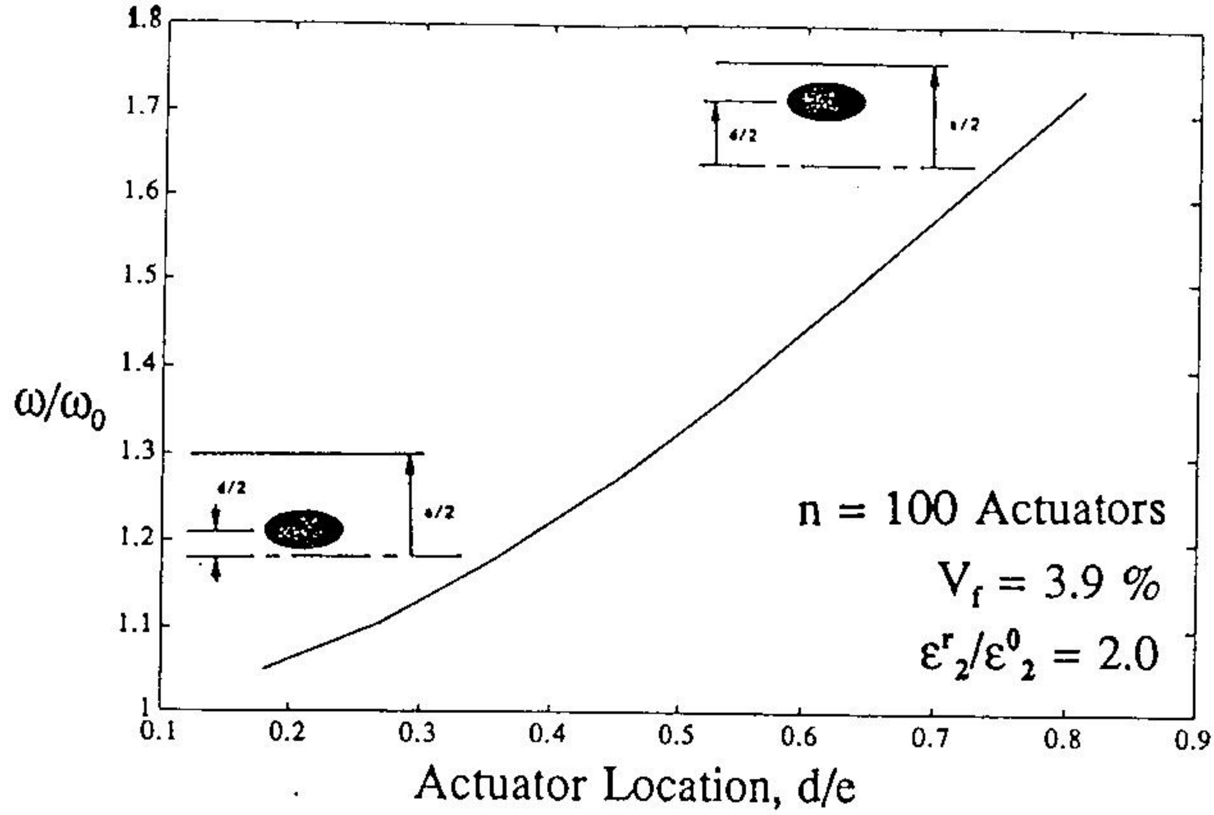

Figure 3. Influence of actuator position on stiffening of the beam.

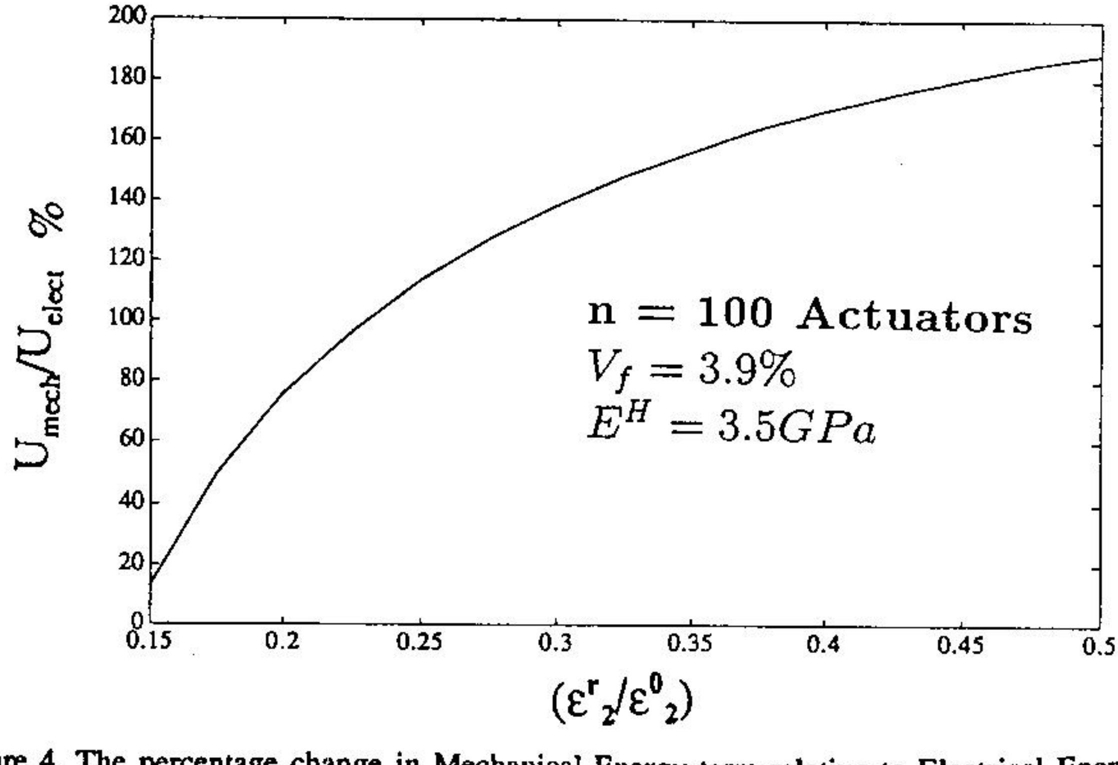

Figure 4. The percentage change in Mechanical Energy term relative to Electrical Energy term as a function of Normalized Actuation strain

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